Encryption

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Contents

Motivation

- Privacy is important for ...
  - **democracy**.
  - the control of our lives.

- Cryptography can be...
  - complex.
  - a lot of **fun**!

Problem

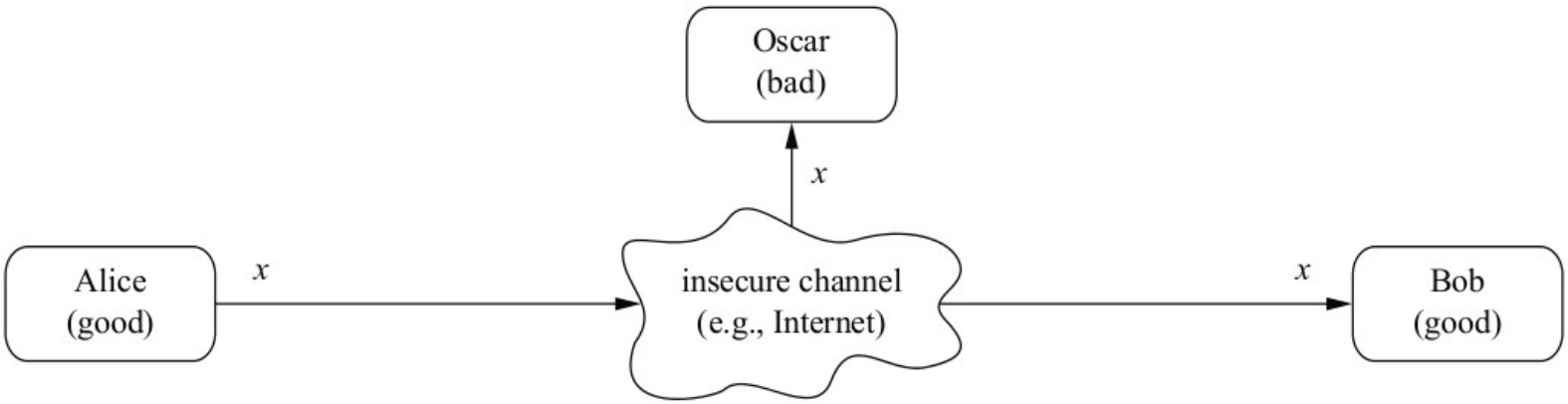

Figure : [26, p.5]

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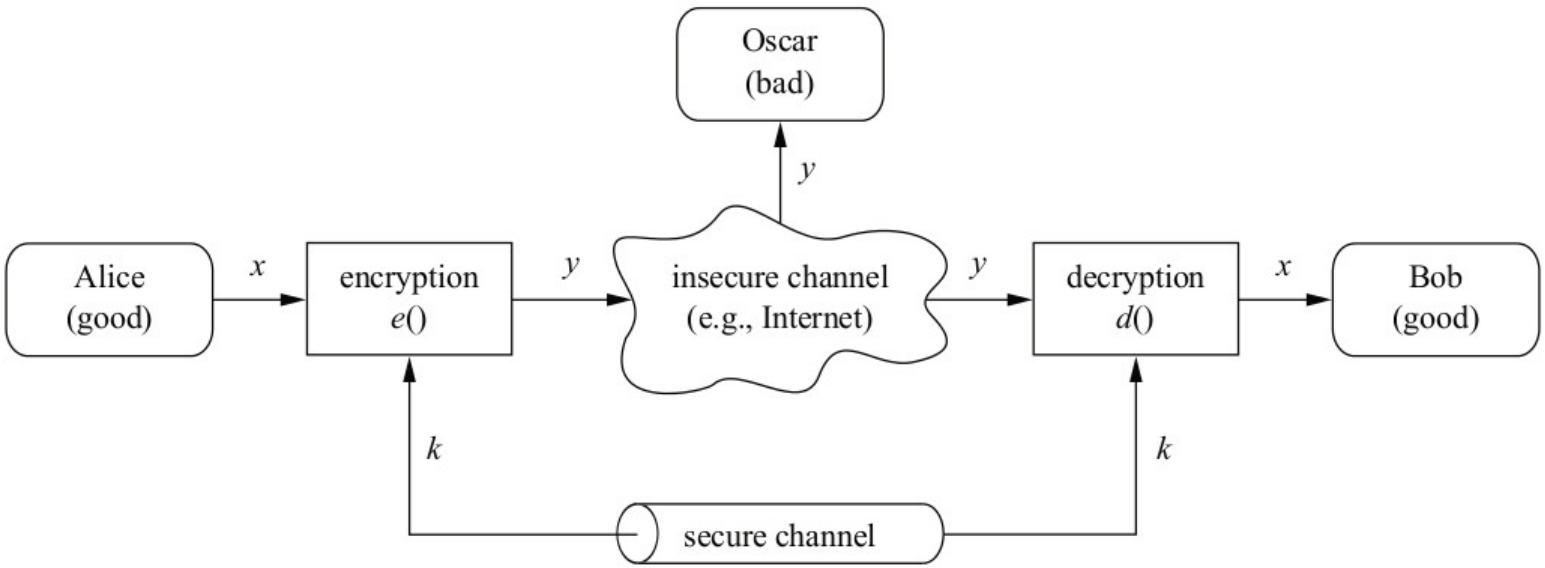

Figure : [26, p.5]

Cryptology

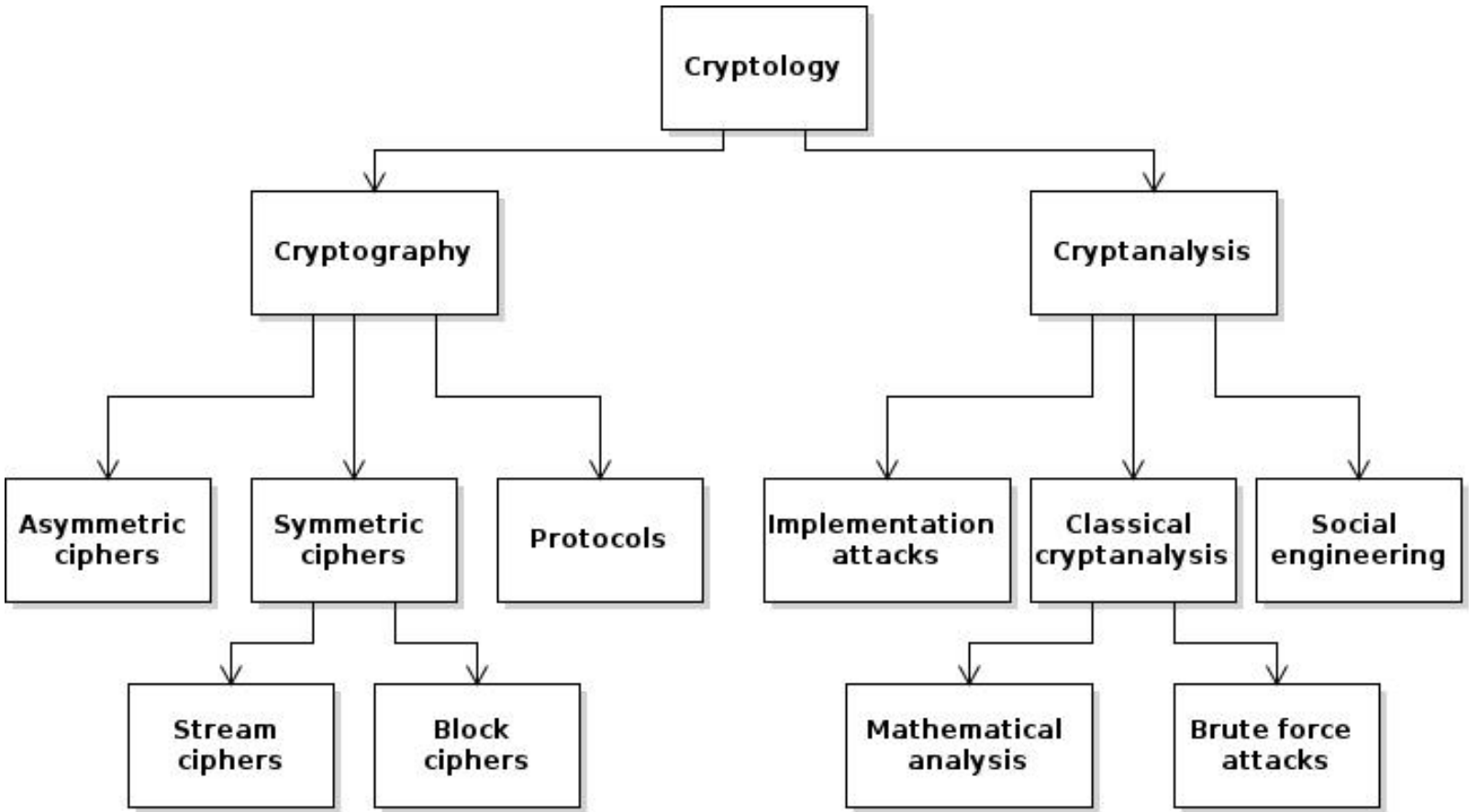

Figure : [26, p.3 and 10]

What is a cipher?

Definition

A cipher defined over $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ is a pair of *efficient* algorithms (E,D) where

$$E : \mathcal{K} \times \mathcal{M} \mapsto \mathcal{C} \qquad \textit{and} \qquad D : \mathcal{K} \times \mathcal{C} \mapsto \mathcal{M}$$

- Efficient: polynomial time
- $\mathcal{M}$: Plain-text space
- $\mathcal{C}$: Cipher- " "
- $\mathcal{K}$: Key space

Question: What is a good cipher?

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Question: What is a good cipher?

Substitution cipher

- Substitute character of the alphabet for another character.
- A particular example: Caesar cipher

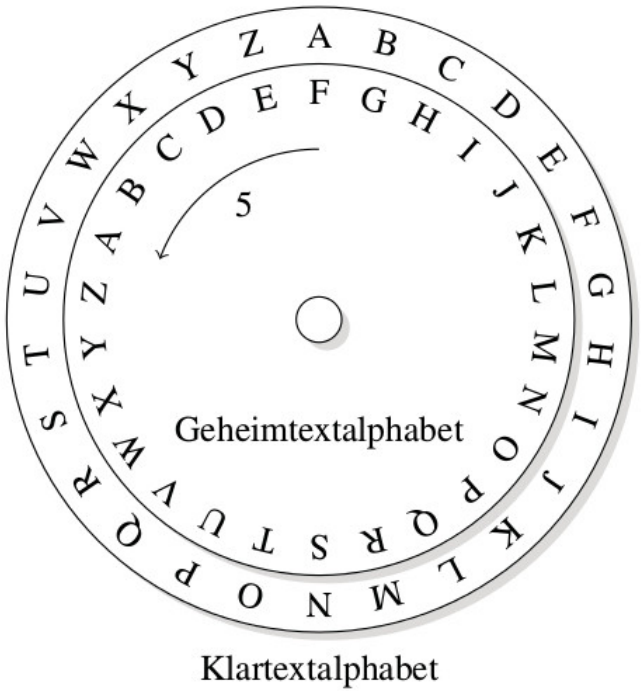

Figure : [26, p.9]

Substitution cipher cont.

- Brute force attack: $|\mathcal{K}| = 26! \approx 2^{88}$

- Very unsecure!

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- Letter frequency attack:

Letter	Frequency
A	0.0817
B	0.0150
C	0.0278
D	0.0425
E	0.1270
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Vigenere cipher

A	0
B	1
C	2
...	...

- Encrypt using modular arithmetic

Example: : $R \rightarrow 17 \quad X \rightarrow 23$

$$(17 + 23) \equiv 40 \equiv 14 \bmod 26$$

Result: $O \rightarrow 14$

- Decryption:

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S	E	C	R	E	T	T	E	X	T

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- Still vulnerable to analytical attacks.

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- Still vulnerable to analytical attacks.
- **Question: Does an invulnerable cipher exist?**

Perfect secrecy

Claude Shanon (1949):

Definition

A cipher (E,D) defined over $(\mathcal{K}, \mathcal{M}, \mathcal{C})$ has perfect secrecy if

$$\forall m_0, m_1 \in \mathcal{M} \quad \forall c \in \mathcal{C} \quad |m_0| = |m_1| :$$

$$\mathcal{P}[\ c = E(k, m_0)\ ] \ = \ \mathcal{P}[\ c = E(k, m_1)\ ]$$

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Perfect secrecy of One-time-pad

One-time-pad has perfect secrecy.
Preliminaries:

$$\mathcal{P}[\ c = E(k,m)\ ] = \frac{|\ \{\ k \in \mathcal{K}\ |\ E(k,m) = c\ \}\ |}{|\mathcal{C}|}$$

$\otimes$: Vigenere encription operation. $\oslash$: V. decription op.

Proof.

For the One-time-pad the following holds:

$$E(k,m) = c \ \Rightarrow\ k \otimes m = c \ \Rightarrow\ k = m \oslash c$$

$$|\ \{\ k \in \mathcal{K}\ |\ E(k,m) = c\ \}\ | = 1 \quad \forall m \in \mathcal{M}\ \forall c \in \mathcal{C}$$

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Perfect secrecy of one time pad cont

- Let cipher-text c be "DFHL". What's the message m?
- Could m be "EVIL", because:

$$"EVIL" \otimes "ZKZA" = "DFHL" \ ?$$

- ... but couldn't m be "GOOD", because:

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m = CIA BIRD

k=PASSWORD

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$m = CI \quad AB \quad IR \quad DX$

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$m = CI \quad AB \quad IR \quad DX$

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Kerckhoff's principle

Kerckhoff's principle:

A cryptosystem should be secure even if the attacker knows all details about the system (except secret key).

DVD content protection

- Used to:
  - protect against piracy
  - enforce regional restrictions
- Streamcipher
- Key length: 40 bits.
- Broken without a brute-force approach.

Famous Symmetric ciphers

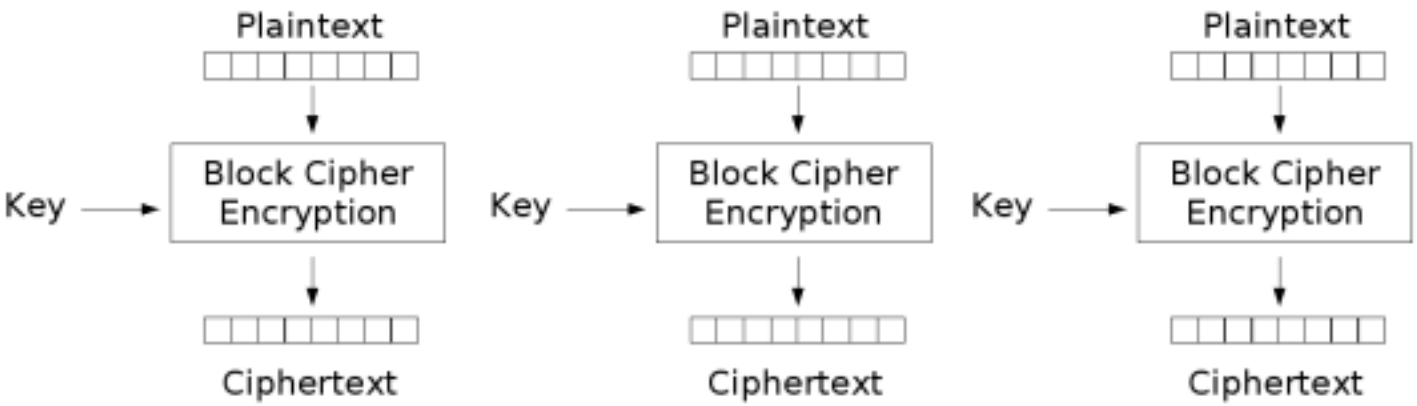
- DES (Data Encryption Standard 1970)
- 3DES (1998)
- AES (Advanced Encryption Standard) 2001
  - RC6
  - Mars
  - Serpent
  - Twofish
  - **Rijandel** $\rightarrow$ **AES**

AES

- Key lengths: 128, 192 or 256 bits.
- Efficient in software and hardware.
- High degree of diffusion and confusion.
- No efficient attacks have been found...
- **...yet!**

Encryption modes

- ECB (Electronic Code Book)

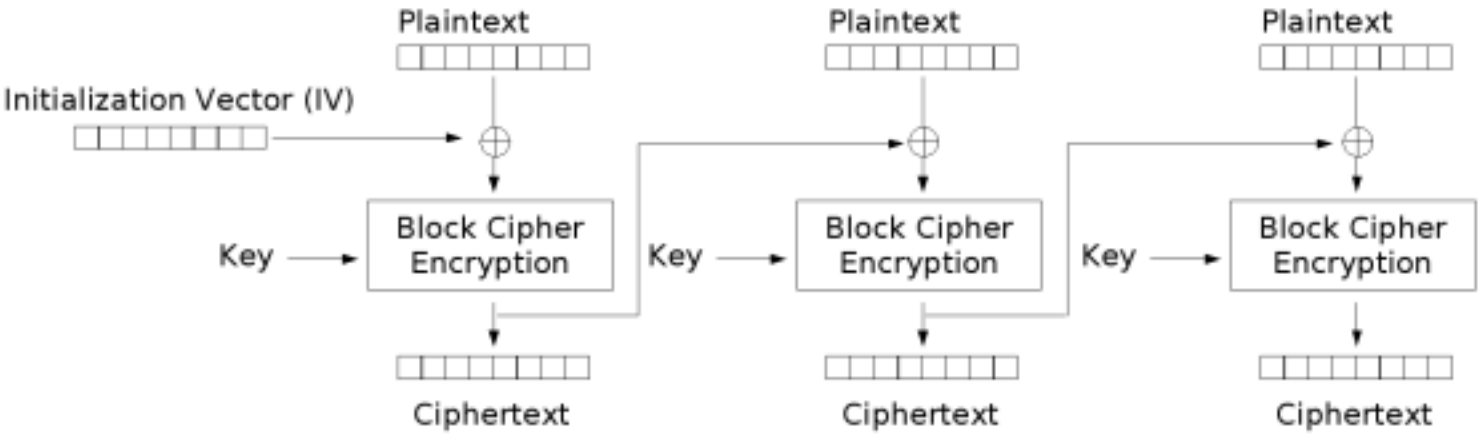


Electronic Codebook (ECB) mode encryption

Figure : [9]

Encryption modes

- CBC (Chipher Block Chaining)



Cipher Block Chaining (CBC) mode encryption

Figure : [8]

Hardware-based encryption

- No performance overhead.
- Transparency.
- HDD/SDDs vendors: disk controller.
- IBMs: Secure Blue
- Encrypt entire boot disk and MBR.

Software-based encryption

- TrueCrypt forks. $\rightarrow$ VeraCrypt
- Bitlocker
- FileVault
- dm-crypt (with LUKS)

Summary

- Kerckhoff's Principle
- Encryption done right depends on:
  - Keyspace
  - Good algorithm (cipher)
  - Implementation
- A secure cipher uses...
  - Confusion
  - Diffusion

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